Phase velocity:- $v = \frac{\omega}{K}$

$$\boxed{v_p = \frac{\omega}{K}}$$

The velocity with which the wave propagates in x-direction. This velocity is called phase velocity of the wave. The phase velocity is also called wave velocity of a monochromatic wave.

Group velocity:-

Now modulating function

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(K)\, e^{i(K-K_0)\left(x - \frac{d\omega}{dK} t\right)}\, dK$$

$$\frac{d\omega}{dK} = \text{group velocity } (v_g)$$

$$\boxed{v_g = \frac{d\omega}{dK}}$$

This is the velocity with which the wave packet travels in a dispersive medium.

we have obtained v_p and v_g.

$$v_p = \frac{\omega}{K} \quad \text{and} \quad v_g = \frac{d\omega}{dK}$$

Now $E = h\nu = \frac{h}{2\pi} \times 2\pi\nu = \hbar\omega$

$$dE = \hbar\, d\omega$$

$$dE = \hbar\, d\omega \quad \text{—(a)}$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda}$$

$$p = \hbar K$$

$$dp = \hbar\, dK \quad \text{—(b)}$$

$$v_g = \frac{d\omega}{dK} = \frac{\hbar\, d\omega}{\hbar\, dK}$$

$$\boxed{v_g = \frac{dE}{dP}}$$

$$v_g = \frac{d\omega}{dK} = \frac{dE}{dP}$$

Now $v_p = \frac{\omega}{K} = \frac{\hbar\omega}{\hbar K}$

$$v_p = \frac{E}{P}$$

$$\boxed{v_p = \frac{\omega}{K} = \frac{E}{P}}$$

Relativistic particle:→

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

$$2E\,dE = 0 + 2p\,dp\,c^2$$

$$2E\,dE = 2p\,dp\,c^2$$

$$\frac{\cancel{2}E\,dE}{\cancel{2}p\,dP} = c^2$$

$$\frac{E}{p}\frac{dE}{dp} = c^2$$

$$\frac{dE}{dp} = c^2 \frac{p}{E}.$$

$$v_g = \cancel{c^2}\frac{\cancel{m}v}{\cancel{m}\cancel{c^2}}$$

$$\boxed{v_g = v.}$$

Thus the group velocity of a wave associated with material particle is same as the particle velocity.

for light

$$\boxed{v_g = c}$$

Relation between v_p and v_g.

$$v_p = \frac{E}{p}.$$

$$v_g = \frac{dE}{dp}.$$

For relativistic particle.

$$\frac{E}{p}\frac{dE}{dp} = c^2.$$

$$\boxed{v_p v_g = c^2}$$